

The Crystal and Molecular Structure of Nitropentamminocobalt(III) Bromide

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The crystal and molecular structure of nitropentamminocobalt(III) bromide, $[\text{Co}(\text{NH}_3)_5\text{NO}_2]\text{Br}_2$, has been solved in order to determine the existence or non-existence of a structural *trans* effect. The crystals are monoclinic (space group $C2/c$, no. 15) with $a=10.71$, $b=8.791$, $c=11.01$ Å and $\beta=94.67^\circ$, with four molecules, each possessing a crystallographic twofold axis, per cell. No significant variation among the five cobalt to ammine nitrogen distances is observed; the other general features of the structure are those expected for the octahedral complex ion.

Introduction

The question of the existence of a *trans* labilizing effect of certain ligands in octahedral complexes has recently come under serious study (Halpern, Palmer & Blakely, 1966). The nitrite ion (coordinated in the nitro form) is one of the more strongly *trans* labilizing ligands in square planar d^8 complexes (Cotton & Wilkinson, 1966). Llewellyn, O'Connor & Odell (1964) have observed, however, that the nitrite ion does not appear to have any kinetically observable labilizing effect on an NH_3 group *trans* to it in the nitropentamminocobalt(III) ion. Since the kinetic data of Halpern *et al.* seem to point to an S_N1 -type mechanism for the replacement reaction involving the NH_3 group *trans* to a ligand $[\text{Co}(\text{NH}_3)_5\text{X}]^{n+}$, a *trans* effect which is attributable to anything other than stabilization of the five-coordinate intermediate by X should be observed as a lengthening of the Co-N bond *trans* to X. We have, therefore, undertaken a study of acidopentammines of cobalt in an attempt to demonstrate the existence or non-existence of what might be termed a 'structural' octahedral *trans* effect. Our general objective is to determine whether there is any correlation between kinetically observed or unobserved *trans* effects and the presence or absence of bond lengthening in the ground state of the reactant complex.

Experimental

Preparation of compound

Crystals of $[\text{Co}(\text{NH}_3)_5\text{NO}_2]\text{Br}_2$ were obtained by allowing a saturated solution of potassium bromide to diffuse through an ultrafine fritted disk into a saturated aqueous solution of $[\text{Co}(\text{NH}_3)_5\text{NO}_2](\text{NO}_3)_2$. Under these conditions crystallization was slow enough that a moderate diffusion rate could be used and crystals suitable for X-ray studies could be obtained within 4–6 hours.

X-ray crystallographic study

The orange-yellow rhombs were examined on a precession camera and were found to belong to the mono-

clinic system with the following cell dimensions: (Mo $K\alpha$, $\lambda=0.7107$) $a=10.71 \pm 0.02$, $b=8.791 \pm 0.015$, and $c=11.01 \pm 0.02$ with $\beta=94^\circ 40'$. The density was measured by flotation in a methylene iodide-carbon tetrachloride mixture as 2.258 g.cm^{-3} which, for a unit-cell volume of 1033 \AA^3 and a formula weight of 349.9 amu, gives 4.01 formula weights per cell.

The only systematic absences observed on the precession photographs were $k \neq 2n$ and $k \neq 2n+1$ for $0kl$ and $1kl$ respectively, giving the general condition for reflection $h+k=2n$, and $l \neq 2n$ for $h0l$, determining the space group as either Cc or $C2/c$.

The intensities of 1324 reflections, $0 < 2\theta < 56^\circ$, were collected on a General Electric XRD-5 manual diffractometer, equipped with a eucentric goniometer head and scintillation counter, using Mo $K\alpha$ radiation. The moving-crystal, moving-counter method (θ - 2θ scan) was used with a 2θ scan range of 2.66° . A crystal approximately 0.2 mm on an edge (equidimensional) was mounted on a glass rod with the b^* axis parallel to the ϕ axis. Data reduction was begun by selecting the statistically acceptable reflections on the criterion that observed intensity (peak-background) must be $\geq 3\sigma$ where $\sigma = \sqrt{\text{peak} + \text{background}}$. The 667 reflections so selected were then corrected for absorption, $\mu=95 \text{ cm}^{-1}$, using the spherically averaged approximation method of Bond (1959) taking μR as 0.7 . Lorentz and polarization corrections were then applied.

Solution and refinement of the structure

A three-dimensional Patterson synthesis was calculated, using the program of Shoemaker, Katz & Seff (1962). Large peaks appeared on the two Harker lines of the type $0, 2y, \frac{1}{2}$ which arise from $C2/c$. It was possible to assign these as Co-Co and Br-Br vectors, assuming the cobalt atoms to lie on the twofold axis and the bromine atoms to be in general positions. Another Br-Br peak appeared on the section $2x, 0, \frac{1}{2} + 2z$ to give a self-consistent solution with the initial positional parameters: Co, $x=0$, $y=0.285$, $z=0.2500$; Br, $x=0.208$, $y=0.002$, $z=0.010$.

One cycle of least-squares refinement (where the function minimized was $\sum \omega ||F_o| - |F_c||^2$) was carried

out with use of the atomic form factors from the *International Tables for X-ray Crystallography* (1962) and allowing the cobalt and bromine positional and isotropic thermal parameters to be varied. The standard R_1 factor ($=\Sigma||F_o|-|F_c||/\Sigma|F_o|$) was then at 0.452. A difference Fourier synthesis was then calculated (Shoemaker, Katz & Seff, 1962) to locate the nitrogen and oxygen atoms. Two more cycles of refinement of all positional and isotropic thermal parameters using the program of Prewitt (1962) gave $R_1=0.124$. At this point an anomalous dispersion correction for cobalt and bromine was applied (*International Tables*, 1962) and full anisotropic refinement was begun. Five cycles of full-matrix least-squares refinement with all reflections given equal (unit) weight brought the residual, R_1 , to 0.08.

A weighting scheme was then introduced so that each F_o was weighted by an amount $\omega=\sigma^{-2}$ where $\sigma=(66.03-3.62|F_o|)^{\frac{1}{2}}$ for $|F_o|\leq 17$, $\sigma=2.72$ for $17<|F_o|<41$ and $\sigma=(0.59|F_o|-15.76)^{\frac{1}{2}}$ for $41\leq|F_o|$. This scheme essentially removed the dependence of $\omega\Delta^2$ on F which had been previously observed. Final convergence of $\sqrt{\Sigma\omega\Delta^2}/(\text{observations}-\text{parameters})$ to a value of 1.058 was obtained. The weighted residual, $R_2=\Sigma\omega||F_o|-|F_c||/\Sigma\omega|F_o|$, had a final value of 0.10. The final calculated structure amplitudes, observed structure amplitudes, $|F_o|$'s and the $|F_c|$'s are listed in Table 1.

A final difference Fourier synthesis showed no peaks higher than about $0.6\text{ e.}\text{\AA}^{-3}$ with a standard deviation, calculated according to the method of Cruickshank (1949), of $0.12\text{ e.}\text{\AA}^{-3}$.

The final fractional coordinates of all atoms in the asymmetric unit are listed in Table 2 with their estimated standard deviations (e.s.d.'s). The e.s.d.'s (σ_j 's) were obtained from the usual least-squares formula, $\sigma_j^2=a_{jj}(\Sigma\omega\Delta^2)/(m-n)$, in which a_{jj} is the diagonal element corresponding to the parameter j in the matrix inverse to the normal equation matrix. Anisotropic thermal parameters, the β_{ij} in the expression $\exp[-(\beta_{11}h^2+\beta_{22}k^2+\beta_{33}l^2+2\beta_{12}hk+2\beta_{13}hl+2\beta_{23}kl)]$, are listed in Table 3.

Table 2. Fractional atomic coordinates and their e.s.d.'s*

	x/a	y/b	z/c
Br	0.2083(2)	0.0144(2)	0.0096(2)
Co	0.0	0.2876(3)	0.2500
N(1)	0.0	0.0631(19)	0.2500
N(2)	0.1834(13)	0.2875(16)	0.2470(12)
N(3)	0.0119(13)	0.2926(15)	0.4278(12)
N(4)	0.0	0.5065(21)	0.2500
O(1)	0.0707(18)	0.5769(16)	0.1878(16)

* E.s.d.'s, occurring in the last significant figure, are given in parentheses.

Table 1. Observed and calculated structure amplitudes multiplied by 48.6

h	k	l	F _{obs}	F _{calc}	h	k	l	F _{obs}	F _{calc}	h	k	l	F _{obs}	F _{calc}	h	k	l	F _{obs}	F _{calc}
0	0	0	1000	1000	0	0	0	1000	1000	0	0	0	1000	1000	0	0	0	1000	1000
1	0	0	1000	1000	1	0	0	1000	1000	1	0	0	1000	1000	1	0	0	1000	1000
2	0	0	1000	1000	2	0	0	1000	1000	2	0	0	1000	1000	2	0	0	1000	1000
3	0	0	1000	1000	3	0	0	1000	1000	3	0	0	1000	1000	3	0	0	1000	1000
4	0	0	1000	1000	4	0	0	1000	1000	4	0	0	1000	1000	4	0	0	1000	1000
5	0	0	1000	1000	5	0	0	1000	1000	5	0	0	1000	1000	5	0	0	1000	1000
6	0	0	1000	1000	6	0	0	1000	1000	6	0	0	1000	1000	6	0	0	1000	1000
7	0	0	1000	1000	7	0	0	1000	1000	7	0	0	1000	1000	7	0	0	1000	1000
8	0	0	1000	1000	8	0	0	1000	1000	8	0	0	1000	1000	8	0	0	1000	1000
9	0	0	1000	1000	9	0	0	1000	1000	9	0	0	1000	1000	9	0	0	1000	1000
10	0	0	1000	1000	10	0	0	1000	1000	10	0	0	1000	1000	10	0	0	1000	1000
11	0	0	1000	1000	11	0	0	1000	1000	11	0	0	1000	1000	11	0	0	1000	1000
12	0	0	1000	1000	12	0	0	1000	1000	12	0	0	1000	1000	12	0	0	1000	1000
13	0	0	1000	1000	13	0	0	1000	1000	13	0	0	1000	1000	13	0	0	1000	1000
14	0	0	1000	1000	14	0	0	1000	1000	14	0	0	1000	1000	14	0	0	1000	1000
15	0	0	1000	1000	15	0	0	1000	1000	15	0	0	1000	1000	15	0	0	1000	1000
16	0	0	1000	1000	16	0	0	1000	1000	16	0	0	1000	1000	16	0	0	1000	1000
17	0	0	1000	1000	17	0	0	1000	1000	17	0	0	1000	1000	17	0	0	1000	1000
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19	0	0	1000	1000	19	0	0	1000	1000	19	0	0	1000	1000	19	0	0	1000	1000
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22	0	0	1000	1000	22	0	0	1000	1000	22	0	0	1000	1000	22	0	0	1000	1000
23	0	0	1000	1000	23	0	0	1000	1000	23	0	0	1000	1000	23	0	0	1000	1000
24	0	0	1000	1000	24	0	0	1000	1000	24	0	0	1000	1000	24	0	0	1000	1000
25	0	0	1000	1000	25	0	0	1000	1000	25	0	0	1000	1000	25	0	0	1000	1000
26	0	0	1000	1000	26	0	0	1000	1000	26	0	0	1000	1000	26	0	0	1000	1000
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57	0	0	1000	1000	57	0	0	1000	1000	57	0	0	1000	1000	57	0	0	1000	1000
58	0	0	1000	1000	58	0	0	1000	1000	58	0	0	1000	1000	58	0	0	1000	1000
59	0	0	1000	1000	59	0	0	1000	1000	59	0	0	1000	1000	59	0	0	1000	1000

Table 3. Anisotropic thermal parameters and their e.s.d.'s* $\times 10^4$

	β_{11}	β_{22}	β_{33}	β_{12}	β_{13}	β_{23}
Br	59(2)	76(2)	58(2)	8(2)	-2(1)	-1(1)
Co	41(3)	53(4)	39(2)	0	-2(2)	0
N(1)	77(20)	34(18)	57(24)	0	33(17)	0
N(2)	47(12)	109(20)	36(10)	9(13)	-2(8)	-17(11)
N(3)	60(12)	83(17)	34(12)	-15(13)	10(9)	-9(11)
N(4)	51(16)	84(22)	25(14)	0	9(11)	0
O(1)	144(22)	99(18)	99(18)	2(18)	44(16)	28(15)

* E.s.d.'s in parentheses.

Results

A projection of the complex ion, $[\text{Co}(\text{NH}_3)_5\text{NO}_2]^{2+}$, onto a plane defined by N(2), N(3) and N(4) is shown in Fig. 1, which also indicates the numbering scheme. The bond lengths and bond angles with their e.s.d.'s are listed in Table 4. A projection of the contents of one unit cell onto the (010) plane is presented in Fig. 2, and pertinent intermolecular contacts (*i.e.*, those ≤ 4.5 Å) are listed in Table 5. The contents of Tables 4 and 5 were obtained using programs written by Wood (1964) and Shoemaker (1963).

Table 4. Bond distances, angles and non-bonded intramolecular contacts

N(1)-Co	1.976 Å	0.019 Å
N(2)-Co	1.985	0.015
N(3)-Co	1.972	0.016
N(4)-Co	1.921	0.021
N(4)-O(1)	1.161	0.022
N(1)-Co-N(2)	89.75°	0.45°
N(1)-Co-N(3)	91.21	0.52
N(2)-Co-N(3)	82.54	0.62
N(2)-Co-N(2')	97.46	0.63
Co-N(4)-O(1)	123.01	0.98
O(1)-N(4)-O(1')	113.9	1.9

Intramolecular contacts

N(2)	N(3)	2.610 Å
	N(3')	2.974
	N(1)	2.794
	N(4)	2.768
	O(1)	2.952
N(3)	N(1)	2.820
	N(4)	2.724
N(3')	O(1)	2.969
O(1)	O(1')	2.114

Table 5. Intermolecular contacts not exceeding 4.5 Å

Co	Br(x, y, z)	4.328 Å
	Br($\frac{1}{2}-x, \frac{1}{2}+y, z$)	4.403
	Br($x, -y, \frac{1}{2}+z$)	4.375
N(1)	Br(x, y, z)	3.625
	Br($x, -y, \frac{1}{2}-z$)	3.544
	N(2) ($x-\frac{1}{2}, y-\frac{1}{2}, z$)	4.152
N(2)	Br(x, y, z)	3.568
	Br($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z$)	3.465
	Br($x, -y, \frac{1}{2}+z$)	3.912
	Br($\frac{1}{2}-x, \frac{1}{2}-y, -(1+z)$)	3.583

Table 5 (cont.)

N(3)	Br($-x, y, \frac{1}{2}-z$)	3.503
	Br($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z$)	3.598
	Br($x, -y, \frac{1}{2}+z$)	3.496
	Br($x-\frac{1}{2}, \frac{1}{2}-y, \frac{1}{2}+z$)	3.836
N(3) ($-x, -y, -z$)	4.001	
N(4)	Br($x-\frac{1}{2}, \frac{1}{2}+y, z$)	3.927
	Br($x-\frac{1}{2}, \frac{1}{2}-y, \frac{1}{2}+z$)	4.403
	N(2) ($x-\frac{1}{2}, \frac{1}{2}+y, z$)	4.175
	N(3) ($x, -y, z-\frac{1}{2}$)	3.981
O(1)	Br($x-\frac{1}{2}, \frac{1}{2}+y, z$)	4.214
	Br($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z$)	3.993
	Br($\frac{1}{2}-x, \frac{1}{2}-y, -z$)	3.440
	N(2) ($-x, y, \frac{1}{2}-z$)	3.850
	N(2) ($\frac{1}{2}-x, \frac{1}{2}+y, \frac{1}{2}-z$)	3.246
	N(3) ($x, -y, z-\frac{1}{2}$)	3.083
Br	N(1) ($x, 1+y, z$)	4.390
	O(1) ($-x, -y, -z$)	4.458
	Br($-x, -y, -z$)	4.458
	Br($\frac{1}{2}-x, \frac{1}{2}-y, -z$)	4.247

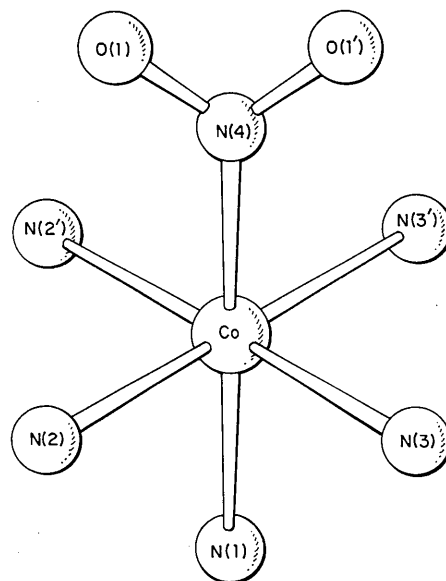
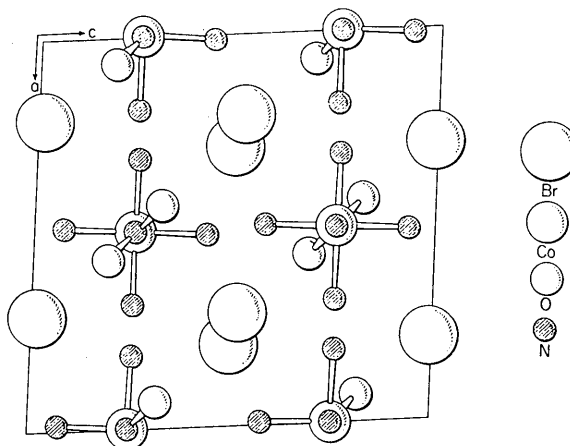
Fig. 1. Projection of the $[\text{Co}(\text{NH}_3)_5\text{NO}_2]^{2+}$ structure on the plane passing through N(2), N(3) and N(4).

Fig. 2. Projection of the unit-cell contents on the 010 plane.

The atoms N(1), Co and N(4) lie on the crystallographic twofold axis and the complex ion has rigorous crystallographic symmetry 2. Its virtual symmetry (neglecting hydrogen atoms) approximates closely to *mm*, with one of the mirrors containing the NO₂ group and bisecting the N(2)–Co–N(3) angle. The substantial difference between the two sets of N–Co–N angles amongst the ligands *cis* to the nitro ligand (82.5° and 97.5°) is, perhaps, surprising. The oxygen atoms of the nitro group lie over the two larger angles and there may be some O···N interaction tending to enlarge these angles. The rather large thermal parameters for the oxygen atoms may contribute to this effect.

There appears to be little evidence of significant hydrogen bonding in this structure, though it is difficult to rule out the existence of some weak hydrogen bonds. A weak one may exist between O and N(3), which lie 3.08 Å. apart. The various N to Br distances (Table 5) are equal to or greater than the approximate sum of van der Waals radii (~3.5 Å) and at least 0.2 Å greater than what might be expected for a fairly normal N–H···Br bond. A check on the angles (Br–N–Br or Br–N–Co) at N(1), N(2) and N(3) showed that few were very close to optimum, though in one case, namely Co–N(3)–Br–(*x*, *y*, $\frac{1}{2} + z$), the angle (109°) is quite satisfactory and the N–Br distance (3.50) is one of the shorter ones.

The most important feature of this structure is the lack of any significant difference between the Co–N(1) bond length and the other four Co–N bond lengths. The mean of value the latter four is 1.978 ± 0.010 Å while Co–N(1) is 1.976 ± 0.019 Å. Evidently the nitro ligand does not significantly weaken the bond *trans* to itself, Co–N(1), relative to the other four which are *cis* to it. In fact, the presence of the nitro ligand does not appear to have any significant influence on the Co–NH₃ bonds at all since the average value of the five bond lengths in this complex, 1.978 ± 0.010 does not differ significantly from the mean of those in [Co(NH₃)₅]³⁺, 1.96 ± 0.02 , as reported by Barnet, Craven, Freeman, King & Ibers

(1966) or the mean of those in [Co(NH₃)₅Cl]²⁺, 1.99 ± 0.01 , as reported by Stanko & Paul (1967).

The absence of any *trans* bond-weakening effect in [Co(NH₃)₅NO₂]²⁺ is in full agreement with the kinetic data of both Halpern, Palmer & Blakely (1966) and Llewellyn, O'Connor & Odell (1964). In the case of [Co(NH₃)₅SO₃]⁺, an appreciable kinetic *trans* effect has been observed (Halpern *et al.*, 1966) and a crystallographic study of a salt of this complex cation has been initiated to see if a 'structural' *trans* effect is present as well.

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